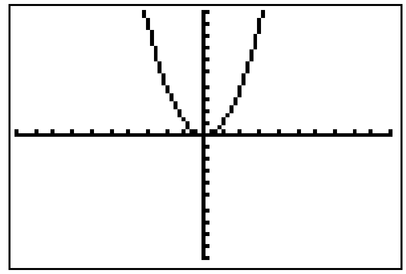
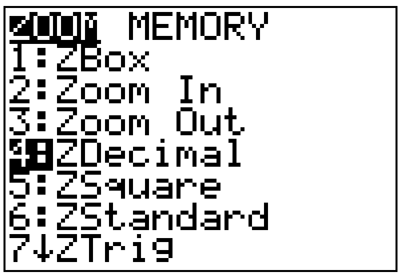
Without Limits – looking at the gradient of the tangent.

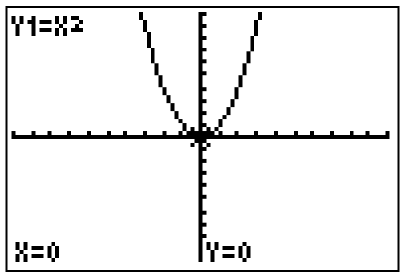
Zooming in on the graph is a way of showing that any curve approximates a linear function, if the interval is small enough.

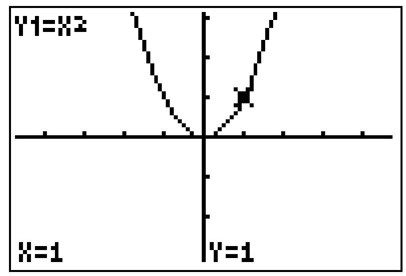
Draw a graph of **$y=x^2$**. Set the Zoom to **Zoom Standard**, then **Zoom Decimal**.

Use **Trace** to put a cursor on the origin, then use the arrow keys to move the cursor along the graph to the point (1, 1).

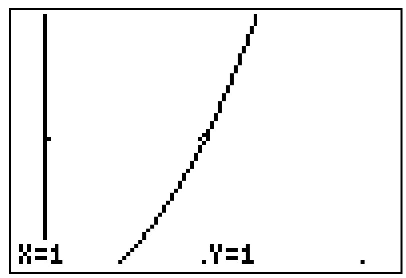








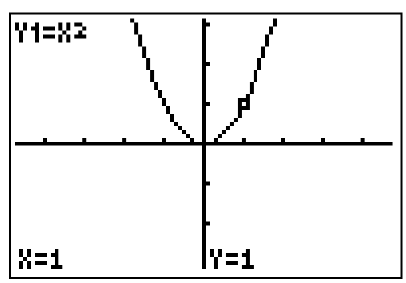
Choose **Zoom In** (then press [ENTER]) and the graph will appear more linear. Check that the cursor is still at (1, 1).

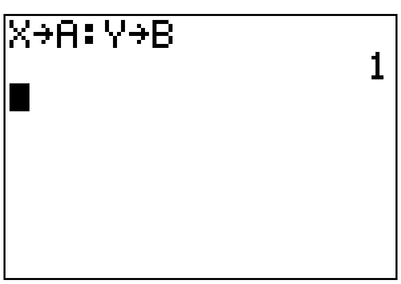


The gradient of a secant approaches the gradient of the tangent as the secant becomes shorter.

To show this, follow these steps:

1. Quit the graph screen by pressing [2nd] [MODE].

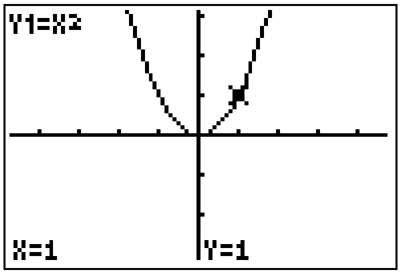


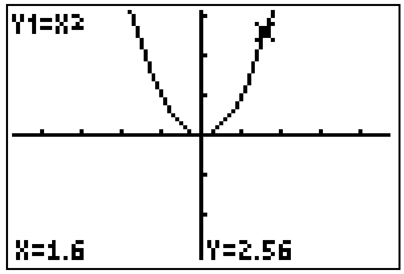


With the cursor on the curve at (1, 1), use the **STO** button to store the initial values for **x** and **y** in memory locations A and B. You **must** use the [X,T,Θ,*n*] for the **x** variable and [ALPHA][1] for the **y** variable note the colon, **:**, is [ALPHA] [.]).

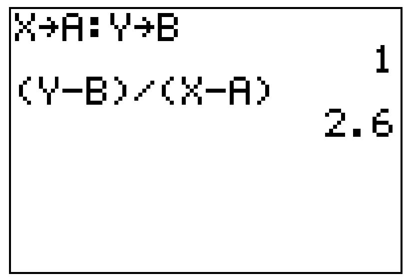
2. Now go back to the graph screen and use **Trace** to move the cursor along to the right of (1, 1). Next you can calculate the gradient of the secant to (1, 1).

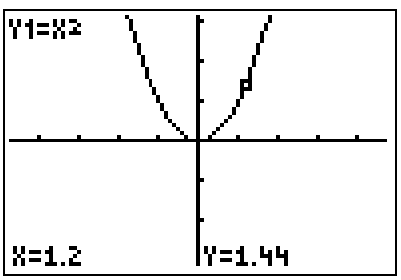
   Press [GRAPH][TRACE] then use [▶] a few times to move the cursor. The calculator now recognizes the new **x** and **y** values.





   Calculate the gradient of the secant as shown on the screen shot. Remember you **must** use the [X,T,Θ,n] for the **x** variable and [ALPHA][1] for the **y** variable. Ask the students to note the gradient of the first secant.

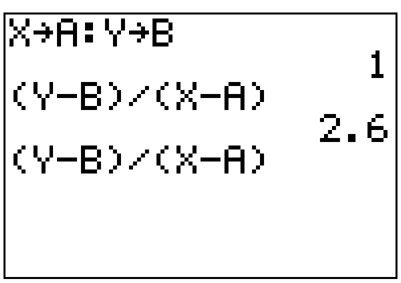


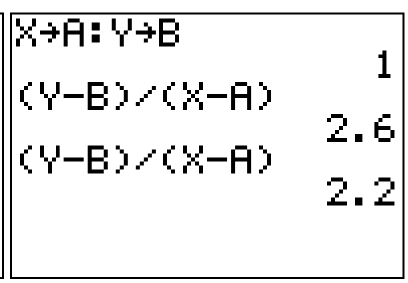


   Return to the graph screen. Use [GRAPH][TRACE] and [◀] to move the cursor closer to (1, 1).

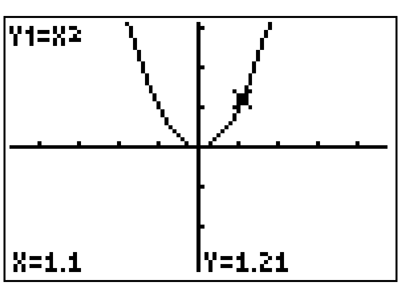
   Calculate the gradient of the new secant. To do this, return to the home screen using [2nd] [MODE] .

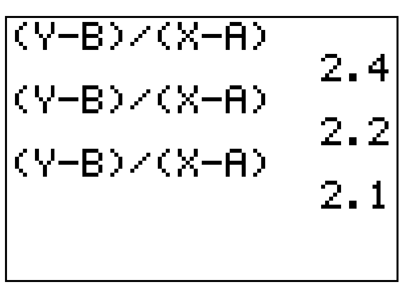
   Note: you do not have to retype the **(Y-B)/ (X-A)** command. Instead, press [2nd][ENTER] and the command is pasted. Press again and the new gradient is calculated.



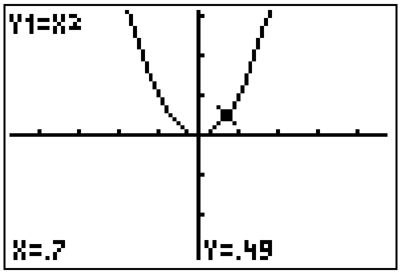


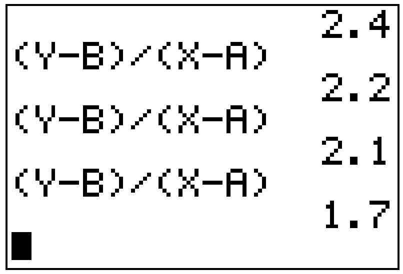
Repeat this process a few more times and ask the student to predict the gradient of $\mathbf{y = x^2}$ at (1, 1).

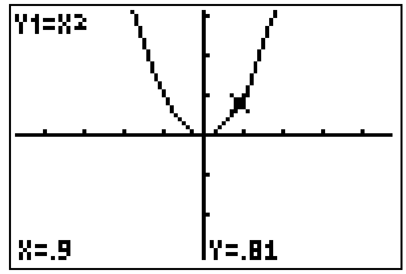


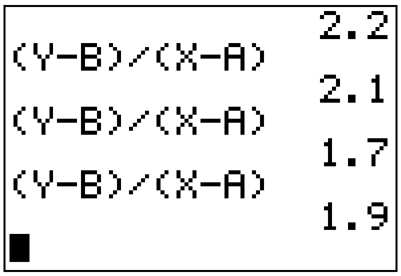


3. They should also approach (1, 1) from below, to see the gradient of the secants also approach the value **m = 2**.

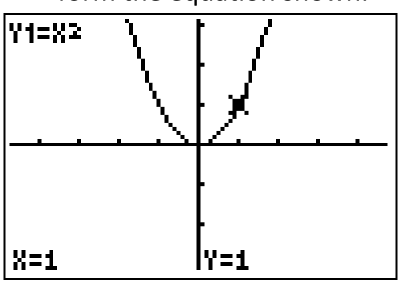


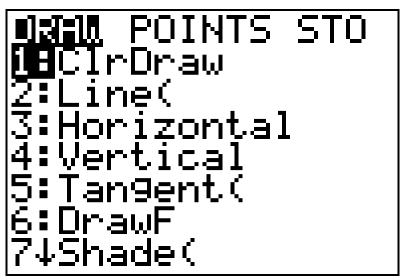


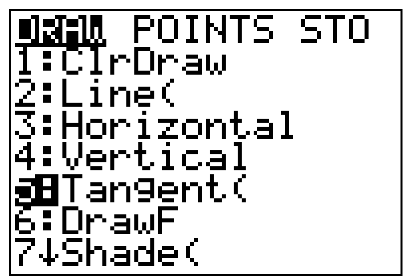


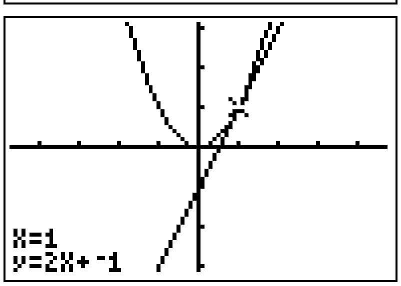


4. The **Draw** command will allow the student to draw the tangent at (1, 1) and observe its gradient form the equation shown.







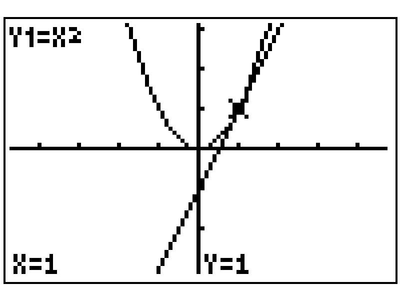


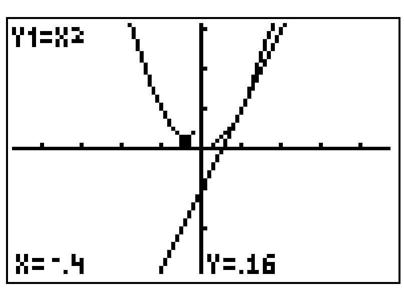
Press [2nd][PRGM] then select **5** to access the Draw Tangent command. Then return to the graph screen and press [ENTER] **two times** for the tangent line and its equation to appear.

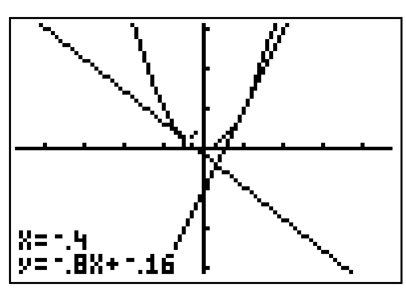
Students should observe that the gradient of the tangent is as expected. **m = 2 when x=1**

The gradient of the tangent depends on the value of x

The student should press [TRACE] again, then move the cursor to a new position and repeat drawing the tangent at the new position. Eventually the students should see the connection between the



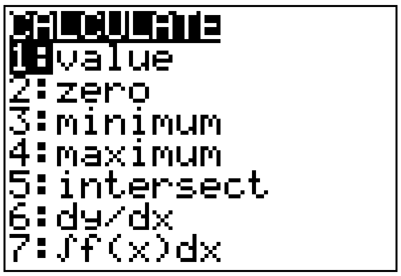


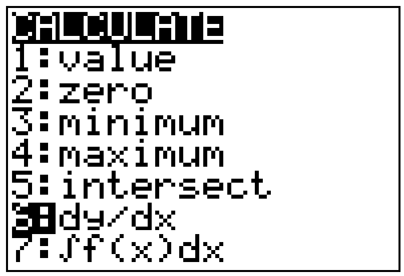


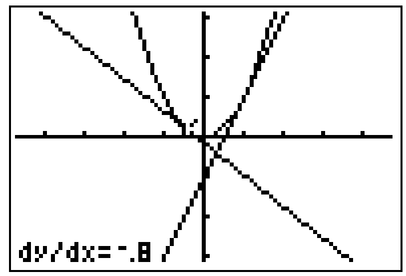
gradient of the tangent and the **x-value** for the function **y=x^2**.

The relationship between the gradient of the tangent and dy/dx at any point can be observed

by using the **calculate dy/dx** command. Have the calculator draw a tangent to the function first if you like.







Press: [2nd][TRACE] first, then **6** to access the **dy/dx** command. Press [ENTER] **two times** and the value of dy/dx appears. The student should observe the relationship between the dy/dx and the gradient of the tangent on their graph.

The general rule dy/dx = n.x^{n-1} for y = x^n can be discovered

by repeating the process for functions other than y=x^2.

Notes on pedagogy

When teaching differentiation it is best that students understand limits first. Newton was correctly criticized for the lack of an adequate foundation for his calculus. The growth of theory on limits provided this much later.

To teach calculus without a foundation is to reduce it to an algorithm.

However we need to recognise that students often fail to understand limits formally. The approach in this exercise allows them to understand limits informally, and thus to still make sense of algebra.